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# Development of invers kinematic method for 6-dof parallel robot using analytical approach

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## Abstract

Study on parallel robot technology has increased over the past six decades. Parallel robot has several advantages compared to serial robot. In term of kinematic aspect, parallel robot has relatively simple calculation compared to serial robot. However, the kinematic calculation is still costly because mostly the studies in parallel robot using numerical based approach. Jacobian based method was widely used to calculate the rotation of actuator required to move the end-effector to the target position and orientation. Therefore, this study proposes an analytical based method to determine the kinematics of 6 DOF parallel robot. The end-effector was attached on the moveable platform, which is connected to the fixed platform using six independent arms. A simulation program based on the proposed algorithm was developed using Matlab. To ensure the implementability of the method, one test using a complex trajectory was performed. The result showed that the proposed algorithm could be used to calculate the rotation angle of every motor to achieve the required position of the end-effector. The implementability of the proposed method to the real 6 dof parallel robot was also tested. The verification test using two trajectories proved that the method was accurate.

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*Keywords:* Parallel robot, end-effector, 6 dof, analytical method

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## 1. Introduction

There are two basic architectures of robot manipulator that widely developed in industrial automation, serial robot and parallel robot. These architectures were categorized based on the kinematics chains that connect the

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output of manipulator to the fixed base. Kinematics chain of serial robot was constructed by a collection of stiff beams. Serial robot has the advantages on the work volume and motion flexibility or dexterity. On the other hand, it has also several weaknesses, such as less precise, low payload ratio, and tend to produce high inertia. Another limitation of serial robot is: the algorithm of invers kinematics for determining the position and orientation of the joints is more complex.

Many studies [1-10] have been performed to develop parallel robot, including the implementation of robot to solve the industrial problems. Khalifa et al [1] developed parallel manipulator to improve endoscopy process. The developed manipulator consists of two motions, rotational and translational motion. Wu et al. [2] developed delta robot with three translational motion and one rotational motion for pick and place operation. Luo and Li [3] constructed 3 degree of freedom (dof) robot with spherical workspace. The robot was called Orthotripod. Another researcher, Ruiz et al. [4] made 3 dof CICABOT robot, which could only perform translation motion using 5 arms. There are many others research on 3 dof parallel robot that have been published [5-10], such as Delta robot [8], 3-UPU [9], and 3-PRC [10], Meanwhile Hendriko et al. [11] developed delta robot 5 dof. Numerous researches on parallel manipulator have been published, however, most of them were 3 dof to 5 dof robot. To improve the flexibility of robot motion and increase the working space, robot with higher degree of freedom is needed.

An effective and efficient method to define the invers kinematics problem is required to obtain a realistic and accurate motions. Many approaches have been developed to solve the kinematics aspect of robot. Mostly proposed numerical based approach, such as Jacobian based approach [12-15] and newton methods [16,17]. These approaches have the disadvantage on the long computational cost.

To overcome the problem of long computational time in numerical based approaches, many research [11, 18-24] have been conducted to produce analytical method in various applications, as well as to compare its computational time to other methods. The results have proven that analytical-based approach is able to decrease the computational time significantly. Hendriko et al. [11] developed analytical method called Analytical Invers Kinematics Simulation (AIKS) for 5 DOF parallel robot. They developed kinematics algorithm for parallel robot with 3 independent legs for linear motion in 3 axis and two additional actuators attached onto the moving platform to rotate the end-effector about the z-axis and the x-axis. From a series of tests, it was found that the method is accurate. A comparison test proved that AIKS was cheaper in term of computational cost than Jacobian method.

Therefore, this study developed a 6 dof parallel robot as depicted in Fig. 1(a). The robot has 6 set of arms in which each of them constructed by upper-arm and lower-arm as shown in Fig. 1(b). Every set of arms was actuated by stepper motor. An analytical algorithm was developed to define the invers kinematics aspect for calculating the orientation and position of end-effector

## 2. Calculating the orientation of the arms

There are six stepper motors were set at the fixed platform and they were used to actuated the upper-arm as shown in Fig. 1(b). The rotation of upper-arm was then move the lower-arm. The orientation of upper-arm and lower-arm were defined using the coordinate of the joints  $A_i$ ,  $B_i$ , and  $C_i$ , where  $i = 1, \dots, 6$ , as shown in Fig. 1(c) and Fig. 1(d). The point  $A_i$  relative to Global Coordinate Frame (GCF) is fixed, that is located at the fixed platform. Because it is fixed, then its coordinate in GCF was known. Meanwhile, the coordinate of  $C_i$  in GCF has been calculated in previous section. Therefore only the coordinate of  $B_i$  that is remaining need to be calculated. For the purpose of simplification the calculation of  $B_i(x_{B_i}, y_{B_i}, z_{B_i})$ , the coordinate of  $C_i$  were transformed to the Joint Coordinate Frame (JCF) at the centered point  $A_i(x_i, y_i, z_i)$  as follow,

$$C_{ij} = [M]_j . C_{iG} ; \quad (i = 1, \dots, 6) \quad (1)$$

$C_{ij}$  is the coordinate of  $C_{iG}$  in JGF, meanwhile  $[M]_j$  is the operator to transform the coordinate system from GCF to JCF, including the rotation about Z-axis ( $\varepsilon$ ) and the centre displacement from E to  $A_i$ . The mapping operator was expressed as follow,

$$[M]_J = Rot(Z, \varepsilon) + A_i = \begin{bmatrix} \cos \varepsilon & -\sin \varepsilon & 0 & x_{A_i} \\ \sin \varepsilon & \cos \varepsilon & 0 & y_{A_i} \\ 0 & 0 & 1 & z_{A_i} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The coordinate  $B_{ij}$  was determined by analyzing two linkages, upper-arm and lower-arm. From the upper-arm's point of view, the distance between  $A_{ij}$  and  $B_{ij}$  can be expressed as follow,

$$L_1^2 = (x_{B_{ij}} - x_{A_{ij}})^2 + (y_{B_{ij}} - y_{A_{ij}})^2 + (z_{B_{ij}} - z_{A_{ij}})^2 \quad (3)$$

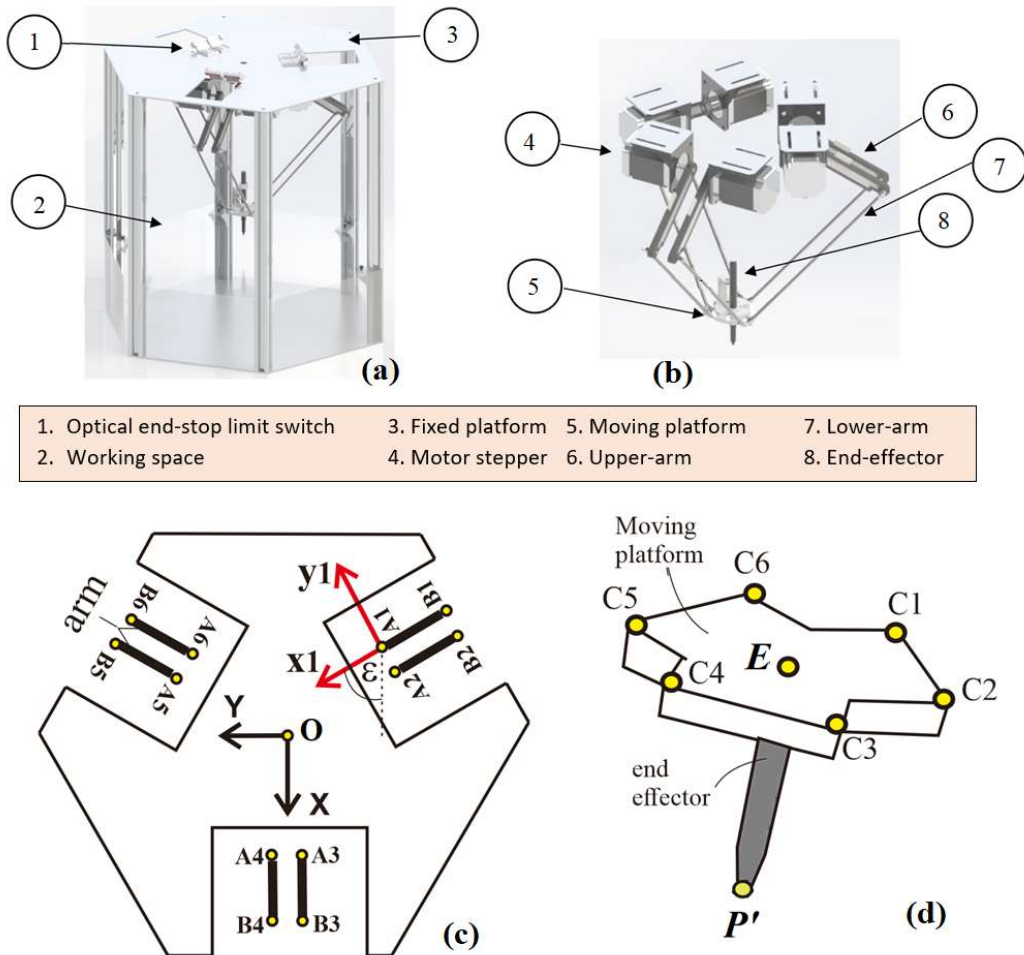


Fig. 1: a) design of parallel robot, b) robot's arms and actuator position, c) end-effector and mobile platform, d) top view of fixed platform

where  $L_1$  is the length of upper-arm. Because the centered of JCF is located at  $A_{ij}$ , then the coordinate of  $A_{ij}$  become  $(0, 0, 0)$ . The  $x$ -axis of JCF at every joint was set align with the upper-arm, as can be seen in Fig. 1c, and hence, the  $y$ -axis of  $B_{ij}$  ( $y_{B_{ij}}$ ) is equal to zero. Then, Eq.(3) yield to become:

$$L_1^2 = x_{B_{ij}}^2 + z_{B_{ij}}^2 \quad (4)$$

There are two unknown variables remaining in Eq. (4),  $x_{B_{ij}}$  and  $z_{B_{ij}}$ . To determine these variables, another equation was required. Therefore, an equation was developed from the lower-arm's point of view. It was expressed as follow,

$$L_2^2 = (x_{C_{ij}} - x_{B_{ij}})^2 + (y_{C_{ij}} - y_{B_{ij}})^2 + (z_{C_{ij}} - z_{B_{ij}})^2 \quad (5)$$

where  $L_2$  is the length of lower-arm. By substituting  $x_{B_{ij}}$  in Eq.(4) into Eq.(5), then Eq. (5) yield to become,

$$T \cdot z_{BS_{ij}}^2 + V \cdot z_{BS_{ij}} + W = 0 \quad (6)$$

where,

$$\begin{aligned} T &= 4 x_{C_{ij}}^2 + 4 z_{C_{ij}}^2 \\ V &= -4 z_{C_{ij}} (L_2^2 - x_{C_{ij}}^2 - y_{C_{ij}}^2 - z_{C_{ij}}^2 - L_1^2) \\ W &= (L_2^2 - x_{C_{ij}}^2 - y_{C_{ij}}^2 - z_{C_{ij}}^2 - L_1^2)^2 - 4 \cdot x_{C_{ij}}^2 \cdot L_1^2 \end{aligned} \quad (7)$$

$z_{B_i}$  in Eq. (6) could be determined using quadratic equation. After  $z_{B_i}$  was obtained, then  $x_{B_i}$  in Eq.(4) could be determined. In this study, the rotation of actuator was represented by the orientation of upper-arm relative to JCF, which is called upper-arm angle ( $\lambda_i$ ). Upper-arm angle is the angle between upper-arm and a vertical line at  $A_i$  ( $\lambda_i$ ), and it was determined as follow,

$$\lambda_i = \cos^{-1} \left( \frac{z_{B_i}}{L_1} \right) \quad (8)$$



Fig. 2: construction of robot and testing on flat trajectory

### 3. Implementation and discussion

The mechanical construction of robot, as presented in Fig. 2, used 4 mm thick aluminium plate for the construction of the roof and the base of robot. While for the pillars, 30x30 mm aluminium rods was used for supporting the base and the roof. The upper-arm was actuated by NEMA 23 stepper motor and they were connected using a bracket. The robot has 6 upper-arms, in which, every upper-arm was connected to a lower-arm using a fish-eye bearing. The arms were used for supporting and moving the mobile platform in carrying the load and the end-effector. Initially, when the start button is pushed, then the robot calibrates the position of end-effector to the home position. The home base means that all the upper-arm at the same orientation. In this case, the upper-arm angle ( $\lambda$ ) was set at  $135^\circ$ . The calibration process was carried out by rotating the stepper motors 1, 3 and 5 counter-clockwise and stepper motors 2, 4, 6 clockwise.

The rotation of every motor was determined using the developed algorithm based on the expected position and orientation of the end-effector. Therefore, the implementability of the proposed method was tested using Matlab. Matlab was used to develop a simulation program based on the derived equation. In this study, the physical of 6 dof parallel robot was also constructed. The motion of the end-effector was determined using the proposed algorithm. Two experimental tests were performed to check the applicability of the algorithm and the accuracy of the robot.

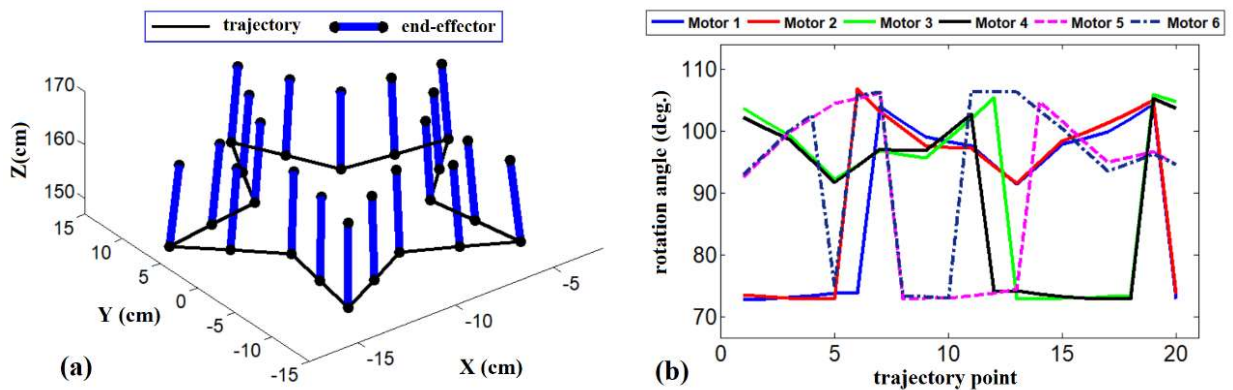


Fig. 3: algorithm implementation test, a) end-effector orientation and position on the designed trajectory, b) rotation angle of every motor

#### 3.1. Calculating the rotation of motor

In this study, the movement of the end effector was actuated using 6 stepper motors. The rotation angle of the motor, which is required to move the end-effector for obtaining the expected position, was calculated using the developed simulation program. To check the applicability of the proposed method, an implementation test was performed. In this test, the end-effector was set to follow a star shape trajectory as shown in Fig. 3a. The blue line with black dots at the end, as presented in Fig. 3a, represented the orientation and position of the end-effector during following the trajectory. There are 20 points in the designed trajectory that were tested. In following the trajectory, all the motors continuously rotated to achieve the expected position and orientation of the end-effector. The rotation angle of every motor was presented in Fig. 3b. From this figure can be concluded that the rotation of motors was very fluctuate due to the posture of end-effector as shown in Fig. 3a.

Verification to ensure the accuracy of the propose algorithm was performed using Solidworks Motion Simulation. Solidworks Motion Simulation are graphical simulations of motion for assembly models. Using this software, the components of robot can be moved to the expected position, and then the position and orientation of the robot can be checked and measured. In this study, the coordinate of point B, which is the joint between upper-arm and lower arm, was checked and compared with those obtained using simulation program. The designed trajectory as shown in Fig. 3a was used for model verification. The results

demonstrated that the data calculated using the proposed algorithm and those measured using Solidworks were exactly similar. The deviations were zero or can be considered as zero. This result could be obtained because this method is analytical, in which the calculation provides exact result. One of the main advantages of analytical based method compared to approximation methods, such as numerical method, is accuracy.

### 3.2. Verifying the accuracy of robot

The accuracy of physical robot in performing the task was tested for two cases. The tests were aimed to check the ability of the end-effector to reach the designed trajectory based on the designed appointed points. For this purpose, a pen was attached on the end effector and a scale paper was put on the flat floor in the working space as shown in Fig. 2. In this test, the robot was programmed to follow two designed trajectories. The trajectory that was obtained by the physical manipulator was then measured and the results were compared with the designed trajectory, as shown in Fig. 4(a) and Fig. 4(b). From these figures can be seen that the deviations were relatively small. The deviation was calculated using a statistical method known as the mean squared error (MSE), and the total error for both tests was 1.71%. Physical checking to the robot was performed, and it was concluded that the deviation was occurred due to the imperfection on the construction of robot, especially on the joint between the upper-arm and lower-arm.

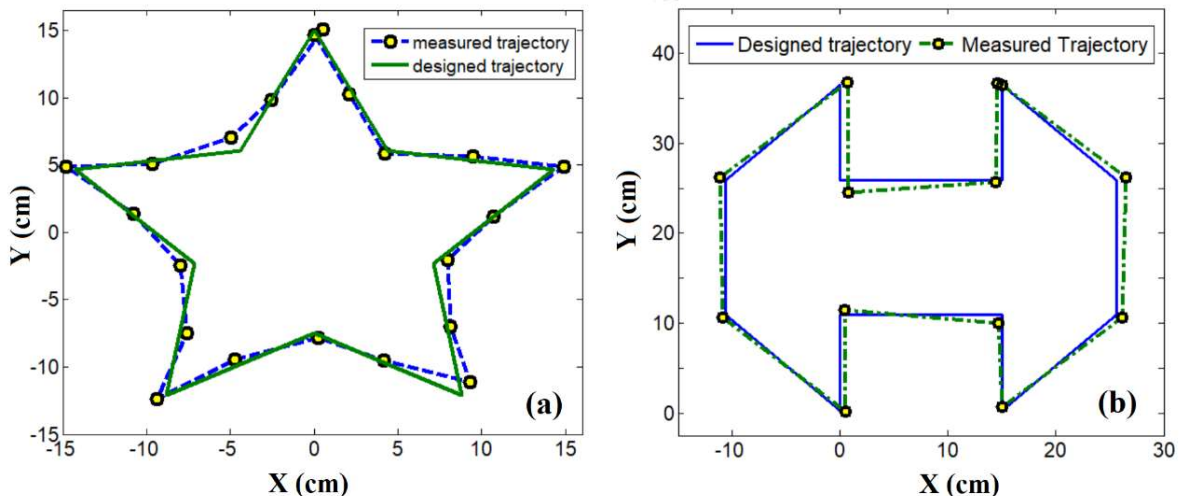


Fig. 4 Experiment test of physical robot on flat surface

In previous study [25], the AIKS method was compared with a Jacobian based method in terms of computational time. The result demonstrated that the computational time of analytical based method was shorter than Jacobian based method. Hence, it can be taken into conclusion that the proposed method is not only accurate, but also computationally cheaper.

## 4. Conclusion

This paper has presented the development of analytical method in calculating invers kinematics of parallel robot. The method of analytical invers kinematics simulation (AIKS) was successful extended so that it could be used for 6 dof parallel robot. Several tests on the implementability of the proposed algorithm, as well as the physical manipulator were performed. Based on the tests result, several conclusions can be taken.

- The implementation test on the analytical method to calculate the rotation degree was performed, the result showed that the algorithm was applicable to calculate the rotation angle of complex trajectory.
- The implementation and accuracy of the physical robot was also tested. The motion of robot was programmed using the proposed algorithm. The robot was tested to follow three different trajectories and the result demonstrated that the robot could follow the trajectories well. The verification tests showed that the deviations of the physical robot were relatively small.

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